THE FLOW OF A COMPRESSIBLE DUST–GAS MEDIUM IN TUBES, IN SEVERAL THERMAL AND STRUCTURAL REGIMES

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We examine the one-dimensional motion of a compressible dust–gas medium in tubes in the case of critical pressure differences and great flow-rate concentrations of a finely dispersed material, with the latter exhibiting various distribution structures in the flow.

The motion of a compressible dust–gas medium in tubes at great pressure differences is characterized by a number of features associated with the critical conditions of the discharge of a two-phase medium, the nature of the distribution of the admixtures in the flow, and with the conditions of the mechanical and thermal interaction between the carrier medium and the admixtures. A particularly significant effect is exerted on the characteristic features of the flow by a change in the structure of the admixture distributions. Motion with a rather uniform distribution of solid particles through the cross section of the tube is found to occur with large inlet velocities for the carrier medium and is characterized by a substantial lag on the part of the solid phase with respect to the carrier medium.

Another case of a structural unique feature in the motion of admixtures for critical pressure differences corresponds to the relatively low velocities of the carrier medium at the inlet to the tube. In this case, as a result of the settling out of the admixtures we find the formation of a "plug" regime which – with an artificial regular and stable motion – exhibits fundamental features that are in good agreement with the requirements of high-pressure pneumatic transport [7].

In this paper we examine two extreme variants of structural organization in a flow. In the first, we assume a uniform distribution for the suspended particles of the admixtures in the lateral cross section of the tube; in the second we deal with a case analogous to "plug" motion, in which the velocity of the carrier medium and that of the admixtures are close in magnitude and differ only as a consequence of the limited filtration of the gas through the packing.

The familiar system of differential equations of motion for a two-phase medium 1-7] – derived through the averaging of integral equations, with all of the assumptions satisfying this averaging and the properties of the hypothetical heterogeneous medium – can be used as the basis, and validly so, for an investigation of the case of one-dimensional flow of a compressible two-phase medium in the presence of finely dispersed admixtures. For the case of one-dimensional flow in a tube we make the assumption that it is possible to calculate the frictional resistance in hydraulic form, and this is in agreement with the assumption as to the linear nature of the variation in the tangential stresses in the lateral cross section of the flow in a tube, for each component of the mixture separately.

The system of equations for the one-dimensional flow of a compressible two-phase medium in tubes can be presented in the following form:

$$\varepsilon\rho u \frac{du}{d\xi} = -\varepsilon \frac{dp}{d\xi} - \frac{\lambda L_*}{2D} \varepsilon\rho u^2 - \varepsilon_1\rho_1 F L_*, \tag{1}$$

$$\varepsilon_1\rho_1 w \frac{dw}{d\xi} = -\varepsilon_1 \frac{dp}{d\xi} - \frac{\lambda_1 L_*}{2D} \varepsilon_1\rho_1 w^2 + \varepsilon_1\rho_1 F L_*, \tag{2}$$

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$$h = c_pT = \frac{1}{k}\left[B - \frac{u_2}{2} - \mu\frac{w^2}{2} - \mu c_1(\theta - T) - L_*(q - r)\right], \tag{3}$$

$$\theta_\sigma - T = -\frac{c_1 m\delta}{\mathrm{Nu}\,\Lambda\sigma L_*} w\frac{d\theta}{d\xi} \equiv -A_1 w\frac{d\theta}{d\xi}, \tag{4}$$

$$\varepsilon\rho uS = \frac{1}{\mu}(\varepsilon_1\rho_1 wS) = G, \tag{5}$$

$$\varepsilon + \varepsilon_1 = 1, \tag{6}$$

$$p = \rho RT = \frac{\rho h(\varkappa - 1)}{\varkappa}, \tag{7}$$

where k is the relative coefficient of heat capacity

$$k = 1 - \mu c_1/c_p + \mu\rho(\varkappa - 1)/\rho_1\varkappa,$$

F is the force of interaction between the gas and the particles [8]

$$F = \frac{24}{\mathrm{Re}_\delta}\left[1 + \varepsilon\,\mathrm{Re}_\delta 10^{-2}(2.59 + \sqrt{1 + (\varepsilon\,\mathrm{Re}_\delta)^{-1}\cdot 2\cdot 10^2})\right]\varepsilon^{-3.75}\frac{f\rho(u - w)^2}{2m},$$

$L_*(q - r)$ is the integral value of the difference between the heat passed through the tube wall and the work of the frictional resistance forces, $\mathrm{Re}_\delta = (u - w)\delta/\nu$.

The values of the resistance factors λ and λ_1 are determined from experimental relationships. For the carrier medium $\lambda = 0.314/\mathrm{Re}^{0.25}$ when $\mathrm{Re} \le 10^5$ while for the transported finely dispersed materials (soot, kaolin) we have $\lambda_1 = (1.0\text{-}1.7)\cdot 10^7(\rho/\rho_1)/\mathrm{Re}^{1.1}$, where $\mathrm{Re} = uD/\nu$ is taken from the velocity of the carrier flow ($\mathrm{Re} \le 10^5$) [7].

The system of equations (1)-(7) is closed and can be modified to a form convenient for investigation. Using (7) and (3) we exclude the pressure derivative from (1) and (2). Assuming $c_p/c_v = \varkappa = \mathrm{const}$, we obtain

$$\frac{dp}{d\xi} = \frac{G(\varkappa - 1)}{\varkappa S}\frac{d}{d\xi}\left(\frac{h}{\varepsilon u}\right). \tag{8}$$

Replacing the derivative $dp/d\xi$ in (1) and (2) from (8) and solving these simultaneously for the velocity derivatives $du/d\xi$ and $dw/d\xi$, we will obtain in dimensionless form

$$\frac{dU}{d\xi} = \frac{L_*}{D}\frac{\Psi_1(U, W) + \Omega_1(U, W, \vartheta')}{\Phi_1(U, W) + \beta\Phi_2(U, W) + \Phi_3(U, W, \vartheta')}, \tag{9}$$

$$\frac{dW}{d\xi} = \frac{L_*}{D}\frac{\Psi_2(U, W) + \Omega_2(U, W, \vartheta')}{\Phi_1(U, W) + \beta\Phi_2(U, W) + \Phi_3(U, W, \vartheta')} \tag{10}$$

and, as a consequence,

$$\frac{dW}{dU} = \frac{\Psi_2(U, W) + \Omega_2(U, W, \vartheta')}{\Psi_1(U, W) + \Omega_1(U, W, \vartheta')}. \tag{11}$$

The heat-transfer equation (4), if we use the enthalpy relationship (3), can also be brought to dimensionless form, i.e.,

$$\frac{d\vartheta}{d\xi} + \frac{\vartheta}{AW\left[1 - \dfrac{(\varkappa - 1)\mu c_1}{\varkappa kR}\right]} = \frac{(\varkappa - 1)H_1}{\varkappa AW\left[1 - \dfrac{(\varkappa - 1)\mu c_1}{\varkappa kR}\right]}. \tag{12}$$

We have introduced the following notation into these equations:

$$U = \frac{u}{\sqrt{RT_0}}, \quad W = \frac{w}{\sqrt{RT_0}}, \quad \vartheta = \frac{\theta}{T_0},$$

$$\Psi_1 = \frac{\lambda U}{2}\left[1 + \mu\left(\frac{\lambda_1 W}{\lambda U} - \beta\right)\Phi_2\right] + \mu\omega\frac{U - W}{W}[1 - \Phi_2(1 + \beta)] + \varphi,$$

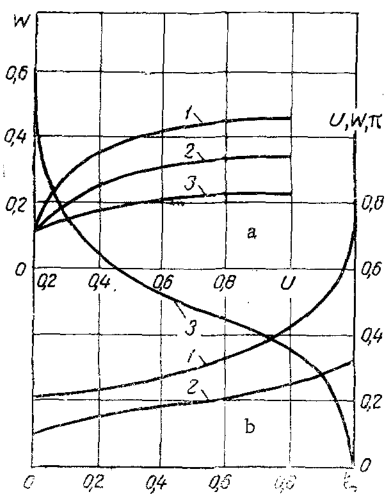

Fig. 1

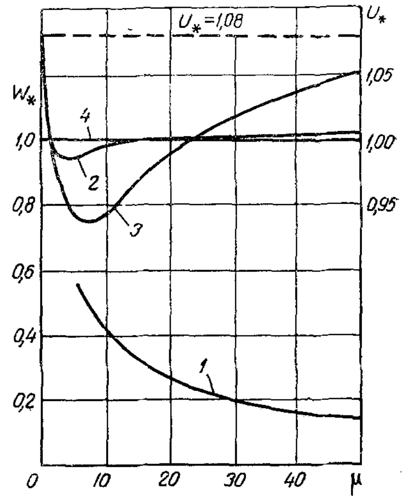

Fig. 2

Fig. 1. a) Dimensionless particle velocity W and gas velocity U for $\delta = 20 \cdot 10^{-6}$ m as a function of μ: 1) $\mu = 10$; 2) 20; 3) 50; b) dimensionless coordinate $\xi = x/L_*$ U (1), W (2), and π (3) for $\delta = 20 \cdot 10^{-6}$ m and $\mu = 20$.

Fig. 2. Critical velocities W_* and U_* as functions of the flow-rate concentration μ: 1) for particles $\delta = 20 \cdot 10^{-6}$ m; 2) for gas with A = 0; 3) the same, for A = ∞; 4) the same, for isothermal flow.

$$\Psi_2 = \frac{\lambda U}{2}\left[(1+\Phi_1)\beta - \frac{\lambda_1 W}{\lambda U}\Phi_1\right] + \omega\frac{U-W}{W}\left[\Phi_1 + (1+\Phi_1)\beta\right] + \varphi\beta,$$

$$\Phi_1 = \frac{\varkappa - 1}{\varkappa k}\left(1 + \frac{H_1 k}{U^2}\right) - 1,$$

$$\Phi_2 = \frac{(\varkappa-1)W}{\varkappa k U}\left(1 + \frac{H_1 k}{W^2}\beta\right),$$

$$\Phi_3 = A\,\frac{(\varkappa-1)\mu c_1 W}{\varkappa k R U}\left(\frac{1}{U} + \frac{\beta^2}{W}\right)\frac{d\vartheta}{d\xi},$$

$$\Omega_1 = A\frac{\mu c_1}{R}\left[\left(1 - \frac{\varkappa-1}{\varkappa k}\mu\beta\right)N\frac{d\vartheta}{d\xi} + \frac{(\varkappa-1)\mu D W}{\varkappa k L_* U}\frac{d^2\vartheta}{d\xi^2}\right],$$

$$\Omega_2 = A\frac{(\varkappa-1)\mu c_1 W}{\varkappa k R U}\left(N\frac{d\vartheta}{d\xi} + \frac{D\beta}{L_*}\frac{d^2\vartheta}{d\xi^2}\right),$$

$$N = \omega\frac{U-W}{UW}(1+\beta) - \frac{\lambda}{2}\left(\frac{\lambda_1 W}{\lambda U} - \beta\right),$$

$$\varphi = \frac{(\varkappa-1)D}{\varkappa k U}\frac{d}{d\xi}\left(\frac{q-r}{RT_0}\right),$$

$$H_1 = \frac{1}{k}\left[\frac{B}{RT_0} - \frac{U^2+\mu W^2}{2} + \frac{L_*(q-r)}{RT_0}\right],$$

$$\omega = \frac{18\eta D}{\rho_1\delta^2\sqrt{RT_0}}\left[1 + \varepsilon\,\mathrm{Re}_\delta\,10^{-2}\left(2.59 + \sqrt{1 + (\varepsilon\,\mathrm{Re}_\delta)^{-1}\cdot 2\cdot 10^2}\right)\right]\varepsilon^{-3.75},$$

$$\beta = \frac{\varepsilon_1}{\varepsilon\mu} = \frac{\rho U}{\rho_1 W} = \frac{G}{\rho_1 W S\sqrt{RT_0} - \mu G},$$

$$A = \frac{c_1\rho_1\delta^2\sqrt{RT_0}}{6\,\mathrm{Nu}\,\lambda L_*},$$

$$\mathrm{Nu} = (2 + 0.495\,\mathrm{Re}_\delta^{\,0.55}\,\mathrm{Pr}^{0.33})(1 - 10\,\varepsilon_1^{0.5}) \qquad (\varepsilon_1 < 3\cdot 10^{-3}).$$

The value of the coefficient A for finely dispersed material can be treated as a constant average value in the light of the insignificant change in the Nu number.

In studying the right-hand member of (11) which is the original equation in the numerical method of the solution, we see that the relationship between the dimensionless values of the velocities W and U is established directly when $A = 0$ or $A \to \infty$ and $q - r = 0$. It is evident that this condition $A = 0$ corresponds to temperature equilibrium between the components of the mixture and is achieved under practical conditions with a very small size for the admixture particles ($\delta \to 0$). Another extreme case $A \to \infty$ occurs with a rather high value for δ or as $L_* \to 0$ and, according to (4), corresponds to the condition $d\vartheta/d\xi = 0$, in which there is no interphase heat transfer. In this special case the numerical solution of (11) leads directly to the relationship $W = f(U)$. On the basis of this relationship we can solve (9), which establishes the dependence of the velocities W and U on the coordinate ξ for the extreme heat-transfer conditions.

The critical condition for the discharge of a two-phase medium corresponds to the singular point on the integral curves of (9) and (10), for which, with the same value of the denominator of the right-hand member equal to zero

$$\Phi_{1*} + \beta\Phi_{2*} + \Phi_{3*} = 0, \tag{13}$$

the derivatives $dU/d\xi$ and $dW/d\xi$ simultaneously tend toward infinity. Equation (13) establishes the relationship between the critical flow parameters, including the relationship between the velocities U_* and W_* [4]. It follows from (11) that the critical value of the derivative $(dW/dU)_*$ remains an extremely limited quantity of the order of β, which indicates substantial lag on the part of the admixture particles from the carrier medium in the outlet critical section of the tube.

Let us examine the motion of a two-phase medium with finely dispersed heavy admixtures for large flow-rate concentrations of the latter, when $\rho/\rho_1 < 0.001$ and $\mu > 5$ and, consequently, β is smaller than 0.0002. To determine the physical features of motion, in this case we use the possible simplification of (9), (11), and (13), assuming that $\beta = 0$.

The critical condition (13) for this flow in substantially simplified and we will have $\Phi_1 = 0$. Expanding the value of Φ_1 and determining the value of the critical particle velocity W_* from this relationship, for $A = 0$ we obtain

$$W = \sqrt{W_0^2 + \frac{1}{\mu}\left[U_*^2 + U_0^2 + \frac{2\varkappa k}{\varkappa - 1}(1 - U_*^2)\right]} \tag{14}$$

In the absence of interphase heat transfer ($A = \infty$) in this relationship we must assume the value of k to be equal to unity. It follows directly from (14) that W_* is a function of the initial values of the velocities W_0 and U_0 and diminishes with an increase in the flow-rate concentration μ. The value of the derivative $(dW/dU)_* = 0$, moreover, indicates that in front of the outlet critical section of the tube the rate of increase in the gas velocity is substantially greater than the rate of increase in particle velocity. The critical tube length L_* in this case, according to (9), will be

$$L_* = D \int_{U_0}^{U_*} \frac{2\Phi_1 dU}{\lambda U + \mu\lambda_1 W\Phi_2 + 2\mu\omega \dfrac{U - W}{W}(1 - \Phi_2)}. \tag{15}$$

We see from the integrand that the critical length diminishes with an increase in the flow-rate concentration μ, with an increase in the dispersion of the admixtures ($\omega \approx \delta^2$), and in weaker fashion as the frictional resistance increases. To illustrate the above-cited points, we undertook the numerical solution of (11) and (9) by the Runge–Kutta method. We examined the flow in a tube with a diameter of $D = 0.027$ m and the initial parameters $U_0 = 0.2$, $W_0 = 0.1$, $T_0 = \theta_0 = 290$°K, $\rho_0 = 0.62$ kg/m^3, $\rho_1 = 180$ kg/m^3, $c_1/c_p = 1$, $\varkappa = c_p/c_v = 1.4$, $\delta = 20\,\mu$, $10\,\mu$, $5\,\mu$, $1\,\mu$, $\mu = 10, 20, 50$, $\beta = \rho_0/\rho_1\mu = < 0.00035$, with air serving as the carrier medium. Figure 1a shows W as a function of U for values of $\mu = 10, 20, 50$ when $\delta = 20\,\mu$. The calculations carried out for the same μ and $\delta \le 20\,\mu$ for various intensities of the interphase heat transfer show the very weak effect of a change in the dispersion of the admixtures (when $\delta \le 20\,\mu$) on the intensity of interphase heat transfer, and of the frictional resistance on the function $W = f(U)$, and in this connection the dispersion remains virtually constant for all variants of flow examined here, and in first approximation is subject to the following relation (when $\mu > 5$):

$$W = W_0 + \frac{1}{\mu}\left\{\left[1 + \frac{\varkappa - 1}{2\varkappa k}\left(U_0^2 - \mu W_0^2\right)\right](UU_0)^{-1} - \left(1 - \frac{\varkappa - 1}{2\varkappa k}\right)\right\}(U - U_0).$$

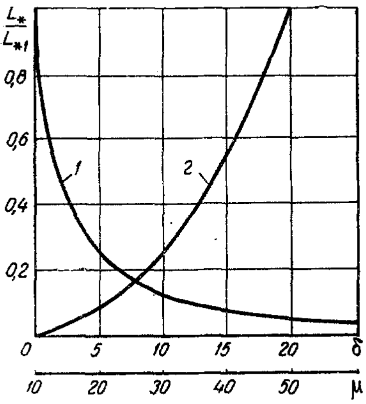

Fig. 3

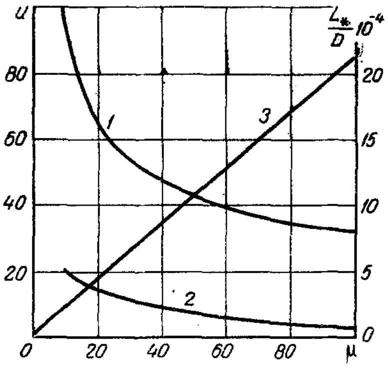

Fig. 4

Fig. 3. Relative critical tube length L_*/L_{*1} as a function of the concentration (1) and of the size (2) of the solid particles. L_{*1} is the critical tube length for $\mu = 10$, $\delta = 20 \cdot 10^{-6}$ m, $U_0 = 0.2$, $W_0 = 0.1$.

Fig. 4. Critical velocity (1), inlet velocity (2), and relative critical tube length L_*/D (3) as functions of the concentration μ for the case of motion without particle lag (u, m/sec).

Figure 1b shows the approximate nature of the dimensionless velocities W, U, and the ratio $\pi = (p - p_*)/(p_0 - p_*)$ as functions of the coordinate ξ. The critical velocities W_* and U_* as functions of the flow-rate concentration μ are given in Fig. 2. They remain virtually unchanged for any intensity of interphase heat transfer, but the initial velocities W_0 and U_0 exert a significant effect. The dimensionless critical gas velocity U_* within the range of variation $10 \leq \mu \leq 50$ in the flow-rate concentration for an adiabatically insulated flow and with an equilibrium temperature for the mixture is close to unity in value, whereas for a homogeneous gas ($\mu = 0$) when $\varkappa = 1.4$, we have

$$U_* = u_* \sqrt{\varkappa} / a_0 = \sqrt{2\varkappa/(\varkappa + 1)} = 1.08.$$

In the absence of interphase heat transfer ($k = 1$), within the same limits for μ, the critical velocity U_* increases with an increase in μ in the limits $0.95 \leq U \leq 0.5$. The critical tube lengths for flows with high concentration of finely dispersed admixtures are extremely limited in magnitude. Thus, for example, for $\mu = 10$ and $\delta = 20\ \mu$ we have $L_*/D = 5.01$ for the above-indicated initial parameters. The nature of the change in L_*/L_{*1} is shown in Fig. 3, from which we see that there is a significant reduction in the critical length as the flow-rate concentration and the dispersion of the admixtures increase. These results are in good agreement with the experimental and theoretical data of [5], but they serve as a significant refinement of the critical discharge parameters for a heterogeneous medium.

The plug regime in the motion of a two-phase compressible medium in tubes is characterized by a velocity lag on the part of the carrier medium with respect to the transported material, in proportion to the filtration of gas through the plug seal formed by the transported material. In this connection, we assume that $w = nu$, where the coefficient n – a function of the filtration rate – is assumed to be constant. Examining the continuous motion of a compressible heterogeneous medium under this condition, on the basis of (1) and (2), we find

$$(1 + \mu n) \frac{du}{d\xi} = -\frac{1}{\varepsilon \rho u} \frac{dp}{d\xi} - \frac{L_*}{2D} (\lambda + n\mu\lambda_1) u. \tag{16}$$

For an adiabatically insulated flow, under the extreme conditions of interphase heat transfer, the enthalpy will be

$$h = \frac{1}{k}\left[B - (1 + \mu n) \frac{u^2}{2}\right]. \tag{17}$$

Eliminating the pressure gradient from (16), we find

$$\left\{1-\frac{\varkappa-1}{\varkappa k}\left[\frac{z}{z-1}+\frac{M^2}{(1+\mu n)(z-1)^2}-\frac{z^2}{2(z-1)^2}\right]\right\}\frac{dz}{z}=\frac{L_*(\lambda+\mu n\lambda_1)}{2D(1+\mu n)}d\xi, \tag{18}$$

where

$$z=\frac{u\rho_1 S}{\mu n G}=\frac{1}{\varepsilon_1}; \qquad M=\frac{\rho_1 S\sqrt{B}}{\mu n G}.$$

Integration of (18) under the condition $z = z_0$ for $\xi = 0$ leads to

$$\left[\frac{(\varkappa-1)M^2}{\varkappa k(1+\mu n)}-1\right]\ln\frac{z}{z_0}-\frac{\varkappa-1}{\varkappa k}\left(\frac{M^2}{1+\mu n}-\frac{1}{2}\right)\left[\ln\frac{z-1}{z_0-1}-\frac{z-z_0}{(z-1)(z_0-1)}\right]$$
$$=\frac{L_*}{2D(1+\mu n)}\int_0^{\xi}(\lambda+\mu n\lambda_1)\,d\xi. \tag{19}$$

The critical tube length is determined from (19) for $\xi = 1$ and $z = z_*$, where z_* is the critical value of this parameter:

$$z_*=1+\sqrt{1+\frac{2M^2(\varkappa-1)-2\varkappa k(1+\mu n)}{[2\varkappa k-(\varkappa-1)](1+\mu n)}},$$

whose value is determined for the limit condition from (18). Correspondingly, the critical value for velocity will be

$$u_*=\frac{\mu n G}{\rho_1 S}\left[1+\sqrt{\frac{2M^2(\varkappa-1)-2\varkappa k(1+\mu n)}{[2\varkappa k-(\varkappa-1)](1+\mu n)}}\right].$$

Characteristic of this flow, given high flow-rate concentrations μ, is the relatively small critical discharge velocity, as well as the large critical tube lengths, and the extremely effective energy balance of transport. To illustrate the above, Fig. 4 shows U_*, U_0, and L_*/D as functions of the flow-rate concentration for the special case: $D = 0.0027$ m, $\mu G = 0.14$ kg/sec, $\rho = 250$ kg/m^3, $p = 9.81 \cdot 10^5$ N/m, $T_0 = 300$°K, and $n = 1$. These results are in good agreement with the experimental data of [6, 7].

In conclusion, we note a substantial difference in the characteristics of motion for compressible heterogeneous media at critical pressure differences, and with large flow-rate concentrations in two separate cases, corresponding to the suspended state of the admixtures and the quasiplug motion of the latter. In the first case the motion is possible only with sufficiently great initial flow velocities and it is characterized by high critical gas velocities, a substantial velocity lag on the part of the admixtures, and extremely limited critical tube length. In the second case, the presence of relatively small critical velocities and larger critical tube lengths is characteristic of the optimum utilization of the gas energy, which leads to the possibility of transporting the admixtures over great distances, at a high flow-rate concentration of the latter [6, 7].

NOTATION

u, w	are the velocities of the gas and of the particles, respectively;
U, W	are the dimensionless gas and particle velocities;
p	is the static pressure;
$\xi = x/L_*$	is the dimensionless coordinate;
L_*	is the critical tube length;
ε, ε_1	are the volume concentrations;
ρ, ρ_1	are the true densities;
D, S	is the tube diameter and the cross-sectional area;
h	is the gas enthalpy;
c_p, c_v, c_1	are the heat capacities of the gas and of the particles;
$\chi = c_p/c_v$;	
μ	is the flow-rate concentration;
T	is the gas temperature;
θ	is the average particle temperature;

δ, m, σ, f	are, respectively, the average particle diameter, its mass, its surface area, and its mid-section;
R	is the gas constant;
Λ	is the coefficient of particle thermal conductivity;
G	is the mass flow rate of the gas.

Symbols

*	denotes the critical parameter;
0	denotes the parameters referred to the inlet section of the tube;
1	denotes the parameters which pertain to the solid phase.

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